

Near Real-Time Risk Simulation of Complex Portfolios on Heterogeneous Computing Systems with OpenCL

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Javier Varela

Summary

I. Overview: Portfolio Risk

II. OpenCL + Workload Allocation

III. Algorithmic Optimizations / Numerical Scheme

IV. Minimizing Device Global Memory

V. Conclusion

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Overview: Portfolio Risk

Portfolio Composition:

- **Stocks** (S_1, S_2)
- **Corporate Bond**
- **Foreign Currency**

- **Options:**
 - European Asian (on S_1)
 - European Barrier (on S_2)
 - American Vanilla (on S_1)
 - American 2D Max (on S_1, S_2)

- **Cash** (no simulation required)



What is the **RISK** at which this portfolio is exposed ?

Overview: Portfolio Risk

What is Value-at-Risk (VaR) ?

„I am α percent certain there will not be a loss of more than **VaR** USD in the next **N** days“ (Hull, *Options Futures and Other Derivatives*).

$$\text{VaR of level } \alpha \quad \text{Portfolio loss function } L \quad \alpha - \text{quantile}$$

$$Var_\alpha(\tilde{L}_{t+N}) \triangleq \inf_{l \in R} \{P(\tilde{L}_{t+N} > l) \leq (1 - \alpha)\} = \inf_{l \in R} \{\tilde{F}_L(l) \geq \alpha\}$$

Loss empirical distribution function $\tilde{F}_L(l) \triangleq P(\tilde{L}_{t+N} > l)$

where α is as high as 95%, 99%, 99.5%.

Industry standard (Basel II and Basel III)

What is Expected Shortfall (ES or cVaR) ?

$$ES_\alpha(\tilde{L}_{t+N}) \triangleq \frac{1}{1 - \alpha} \int_{\alpha}^1 Var_\gamma(\tilde{L}_{t+N}) d\gamma$$

Overview: Portfolio Risk

How to compute Value-at-Risk (VaR) ?

- 1. Historical simulation:** describes future changes ...
based on empirical distribution of observed past data.
- 2. Variance-Covariance method:** first-order approximation of the loss function L.
assumes normally distributed returns.
- 3. Monte Carlo (MC) method:** simulates the loss function L.
 - ✓ Does not rely on historical data.
 - ✓ No need for approximation.
 - ✓ No assumption of a normal distribution.
 - ❖ Computationally intensive problem
 - ❖ Heterogeneous problem
(many different algorithms involved)



Exploit OpenCL

Overview: Portfolio Risk

Portfolio Simulation:

- Stocks (S_1, S_2)
- Corporate Bond
- Foreign Currency

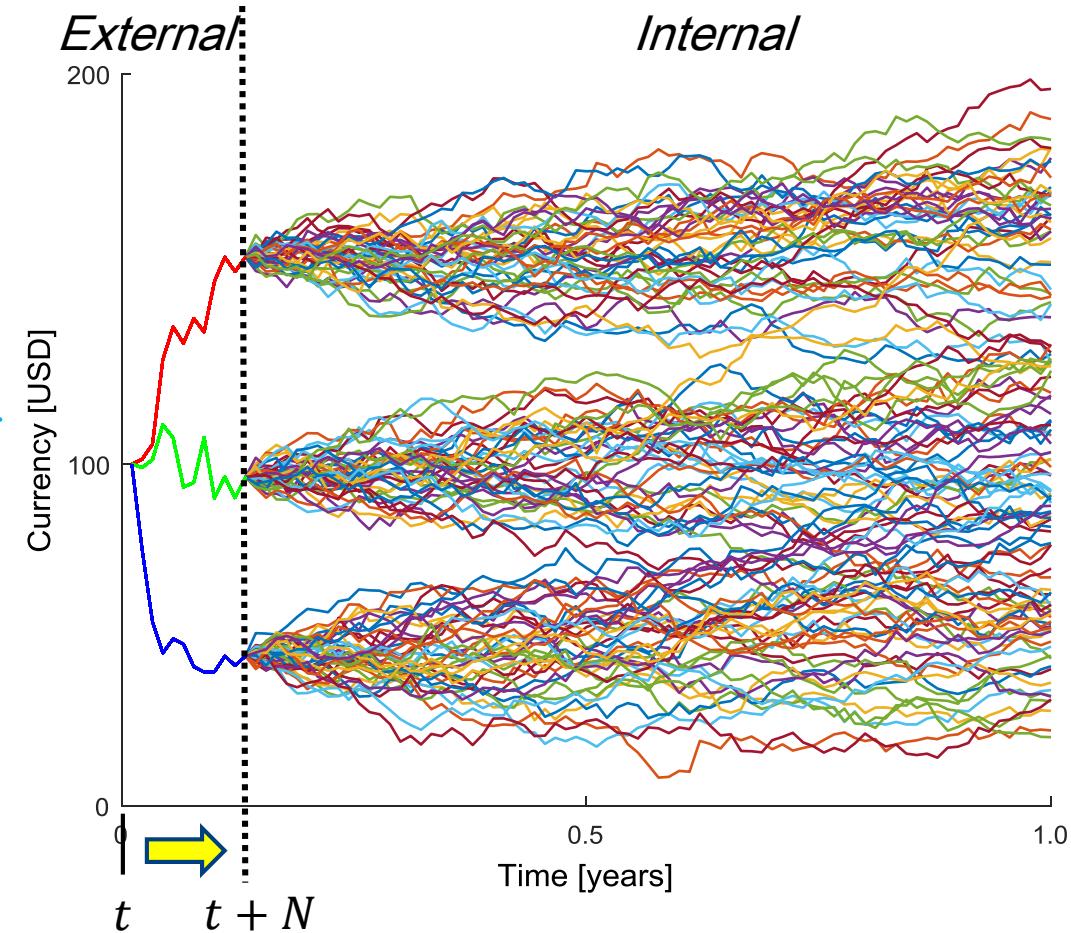
Options:

- European Asian (on S_1)
- European Barrier (on S_2)
- American Vanilla (on S_1)
- American 2D Max (on S_1, S_2)

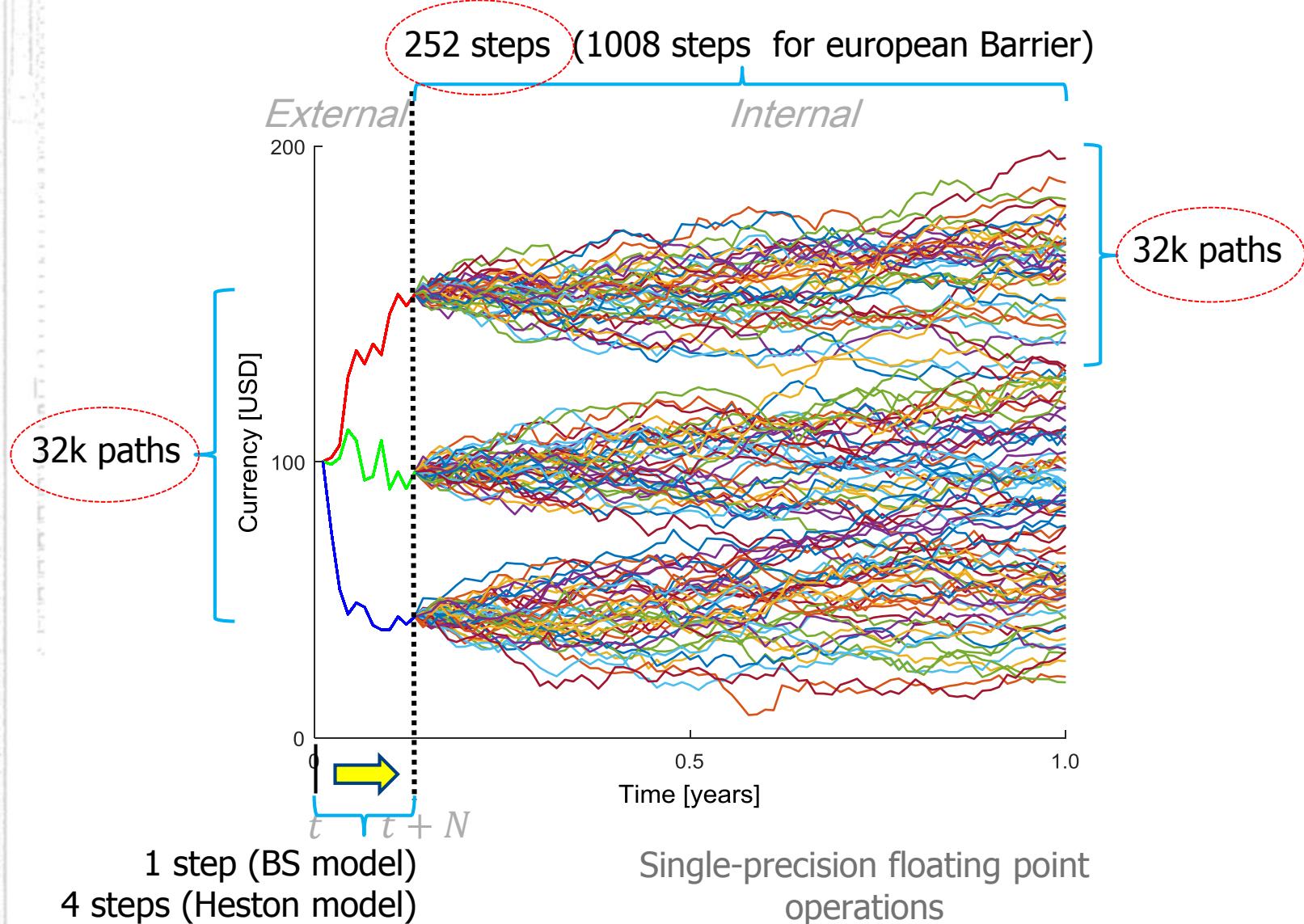
➤ Cash (no simulation required)

Two Models:

- Black-Scholes (BS)
- Heston

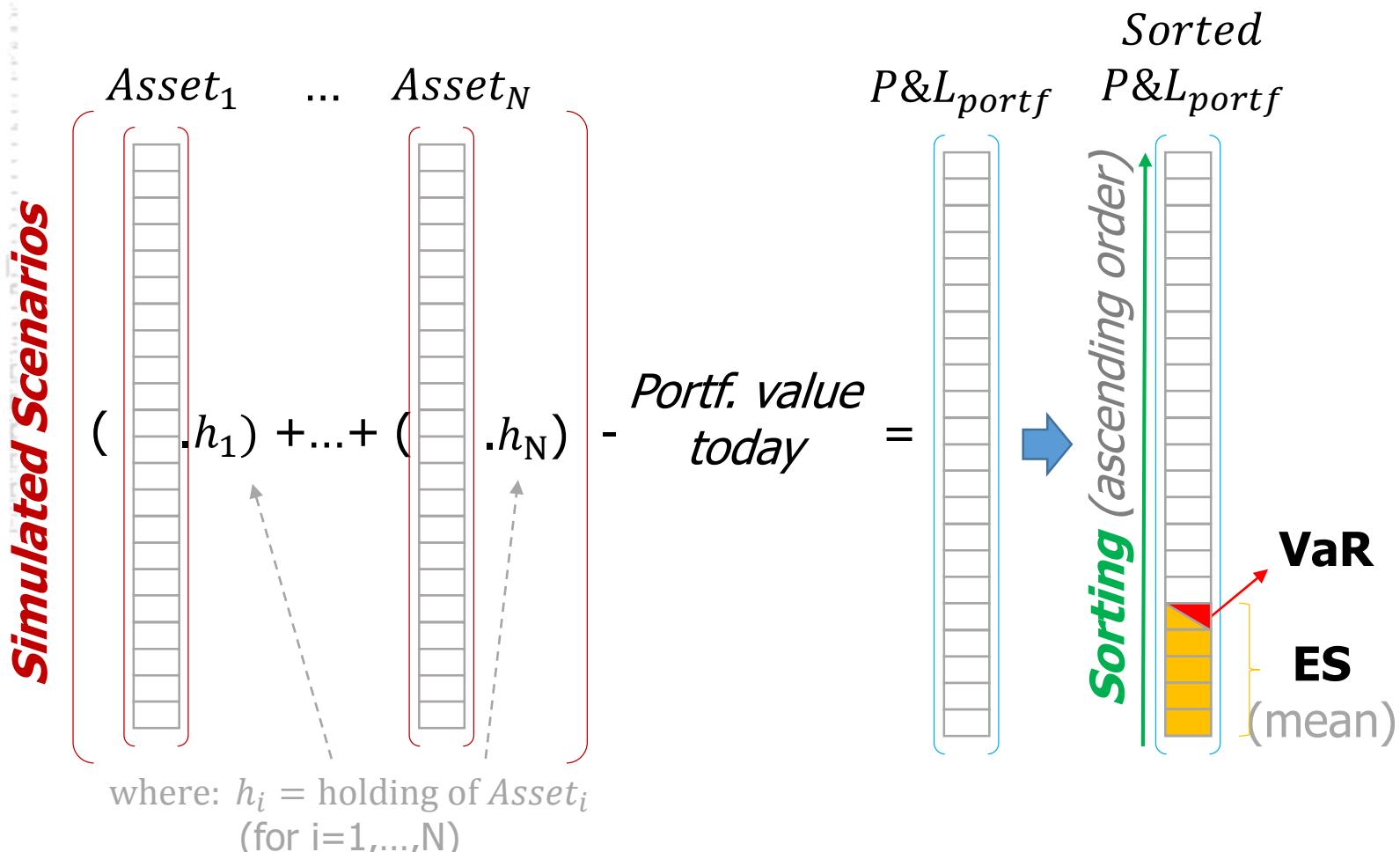


Overview: Portfolio Risk



Overview: Portfolio Risk

Sequence of steps:



Summary

I. Overview: Portfolio Risk

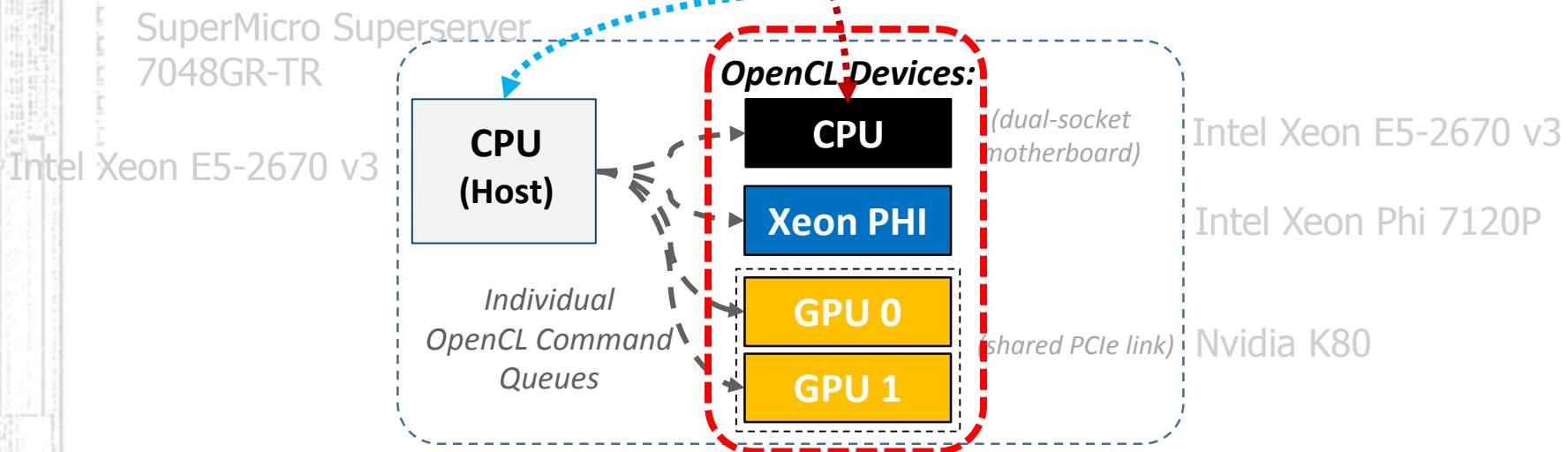
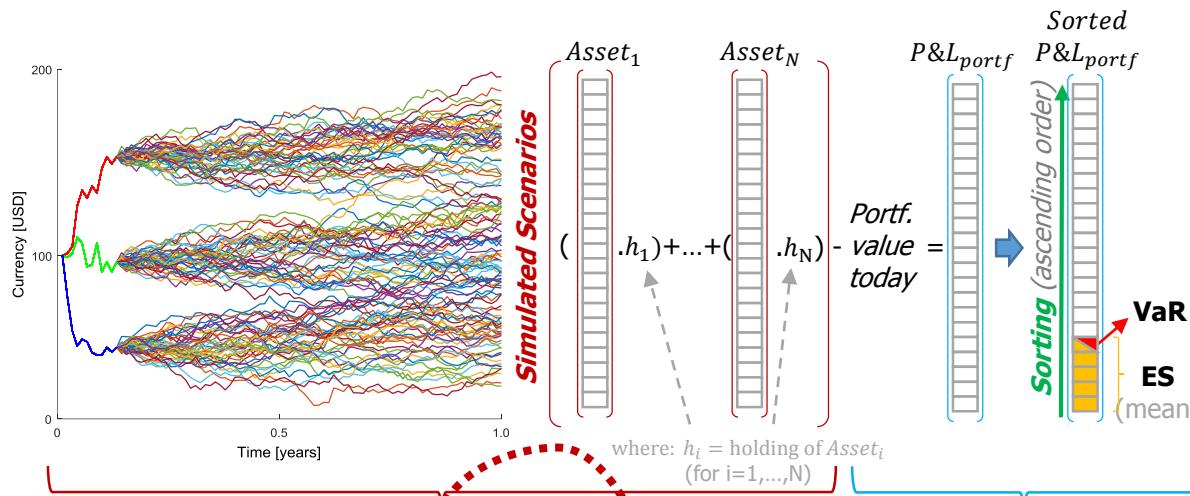
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OpenCL + Workload Allocation



OpenCL + Workload Allocation

ILP formulation

(Matlab's *intlinprog*)

$$\min_x f^T \cdot x \text{ subject to} \begin{cases} x_i \text{ are integers} \\ A \cdot x \leq b \\ A_{eq} \cdot x \leq b_{eq} \\ lb \leq x \leq lu \end{cases}$$

Set of Kernels:

$$K = \{k_{Stocks2Corr}, k_{Bond}, k_{F.Currency}, \dots\}$$

Set of Devices:

$$D = \{d_{CPU}, d_{PHI}, d_{GPU0}, \dots\}$$

Binary decision variables: $x_{k,d}$, where: $0 \leq x_{k,d} \leq 1$

Total runtime: x_T , where: $0 \leq x_T \leq \infty$

OpenCL + Workload Allocation

ILP formulation ... (continued)

Constraints:

Single assignment of each kernel among all devices:

$$\forall k \in K, \sum_{d \in D} x_{k,d} = 1$$

Device runtime:

$$\forall d \in D, \sum_{k \in K} t_{k,d} \cdot x_{k,d} - x_T \leq 0$$

Device global memory:

$$\forall d \in D, \sum_{k \in K} m_{k,d} \cdot x_{k,d} \leq M_d$$

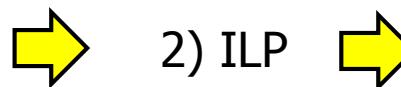
Cost function:

$$f^T \cdot x = x_T$$

OpenCL + Workload Allocation

Workload Allocation with ILP

1) Profile each kernel
on every device



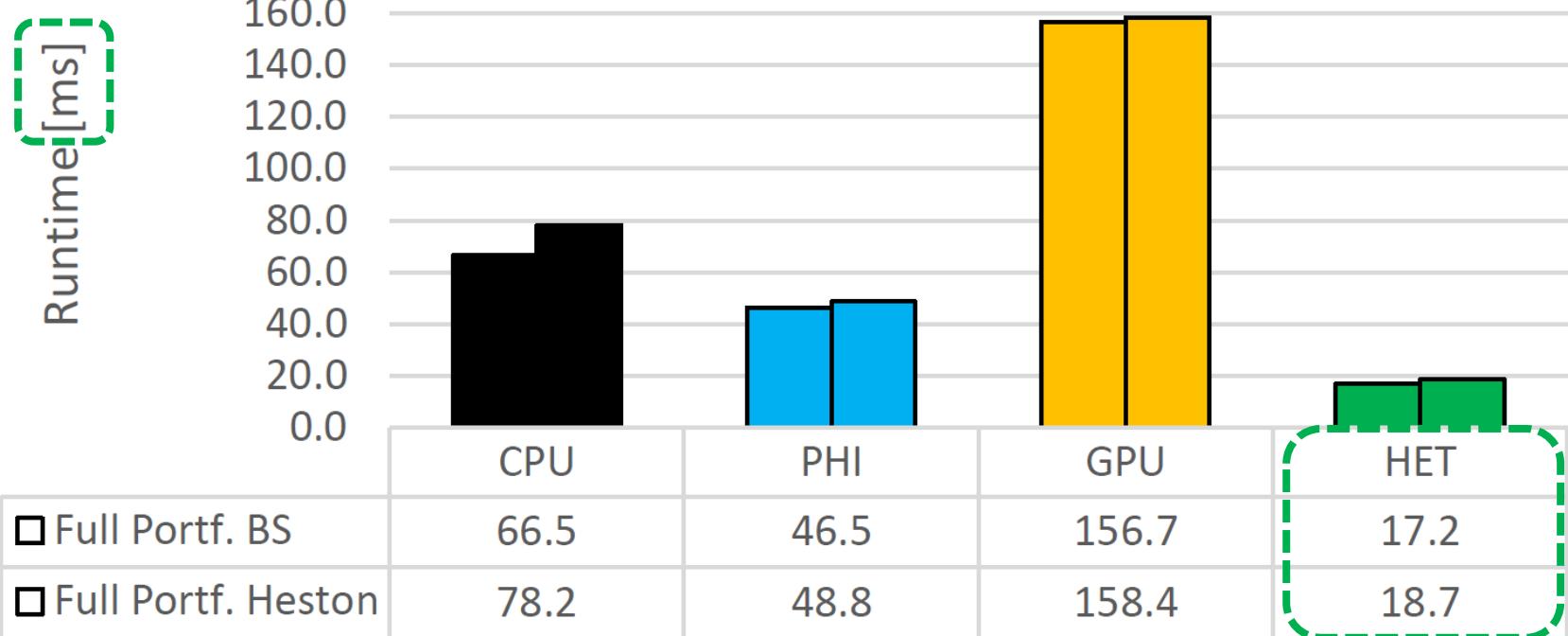
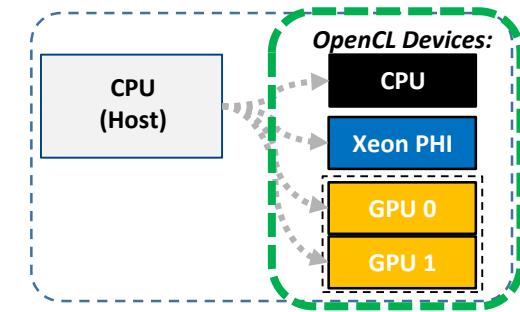
2) ILP

| Kernel | Model | Runtime | | |
|--|--------|----------|----------|---------|
| | | CPU | PHI | GPU |
| <i>External Simulation</i> | | | | |
| Stocks2Corr | BS | 0.12 ms | 0.28 ms | 0.02 ms |
| | Heston | 0.15 ms | 0.43 ms | 0.03 ms |
| Bond | - | 0.22 ms | 0.37 ms | 0.05 ms |
| F.Currency | - | 0.04 ms | 0.30 ms | 0.01 ms |
| <i>Internal Simulation: European Options</i> | | | | |
| Asian(RNs) | BS | 8.89 s | 6.59 s | 6.74 s |
| | Heston | 17.39 s | 11.43 s | 11.09 s |
| Asian(InfoP1) | BS | 2.83 ms | 1.14 ms | 0.28 ms |
| <i>2DMax(RNS)</i> | | | | |
| 2DMax(PathsP1) | Heston | 399.24 s | 572.37 s | 91.92 s |
| | BS | 14.31 ms | 13.78 ms | 0.99 ms |
| 2DMax(PathsP2) | Heston | 19.02 ms | 20.52 ms | 1.45 ms |
| - SurfGen | - | 65.33 s | 47.78 s | 41.20 s |
| | - | 78.89 s | 54.93 s | 48.11 s |
| 2DMax.Interp | - | 0.05 ms | 0.32 ms | 0.01 ms |

| Kernel | CPU | PHI | GPU0 | GPU1 |
|--|-----|-----|------|------|
| Model: BS | | | | |
| Stocks2Corr | - | - | - | ✓ |
| Bond | - | - | - | ✓ |
| F. Currency | - | - | - | ✓ |
| Euro. Asian (<i>Info-reuse</i>) ¹ | - | - | ✓ | - |
| Euro. Barrier (<i>Info-reuse</i>) ¹ | - | ✓ | - | - |
| Amer. Vanilla. BT | ✓ | - | - | - |
| Amer.2DMax (<i>Interp+S.Load</i>) | - | - | - | ✓ |
| Model: Heston | | | | |
| Stocks2Corr | ✓ | - | - | - |
| Bond | ✓ | - | - | - |
| F. Currency | - | - | ✓ | - |
| Euro. Asian (<i>Info-reuse</i>) ¹ | - | - | - | ✓ |
| Euro. Barrier (<i>Info-reuse</i>) ¹ | - | ✓ | - | - |
| Amer. Vanilla. BT | ✓ | - | - | - |
| Amer.2DMax (<i>Interp+S.Load</i>) | - | - | - | ✓ |

OpenCL + Workload Allocation

Kernels Runtime



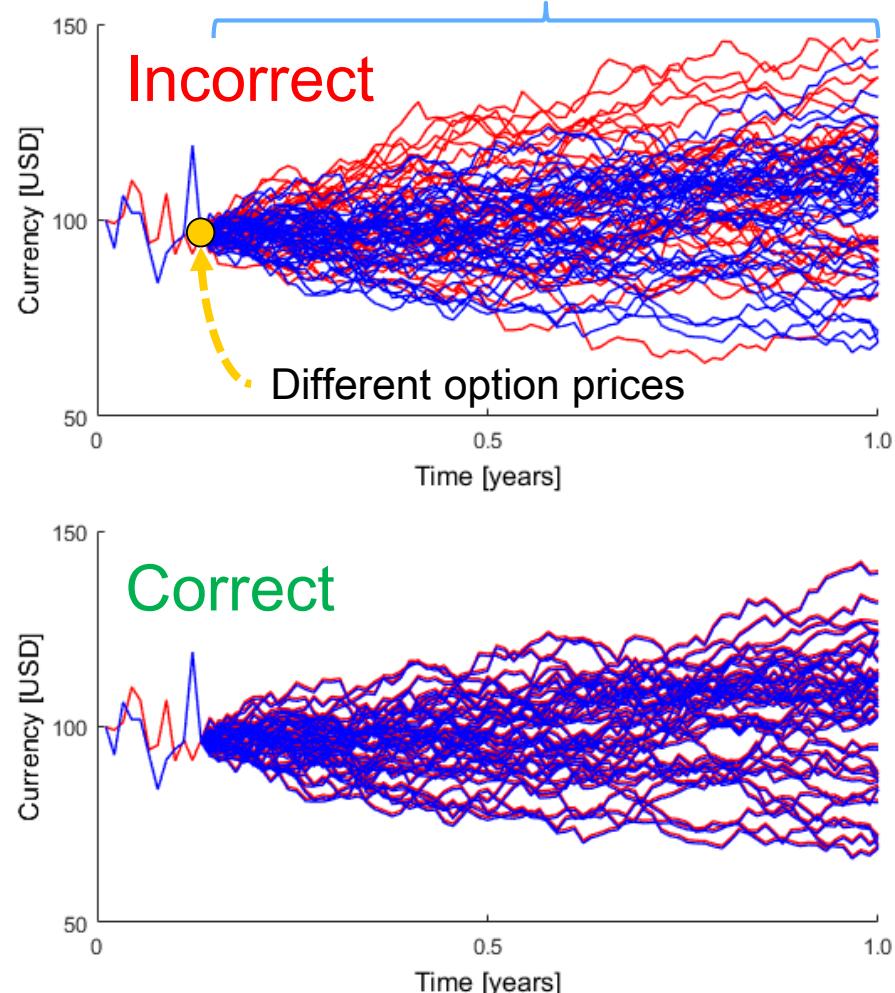
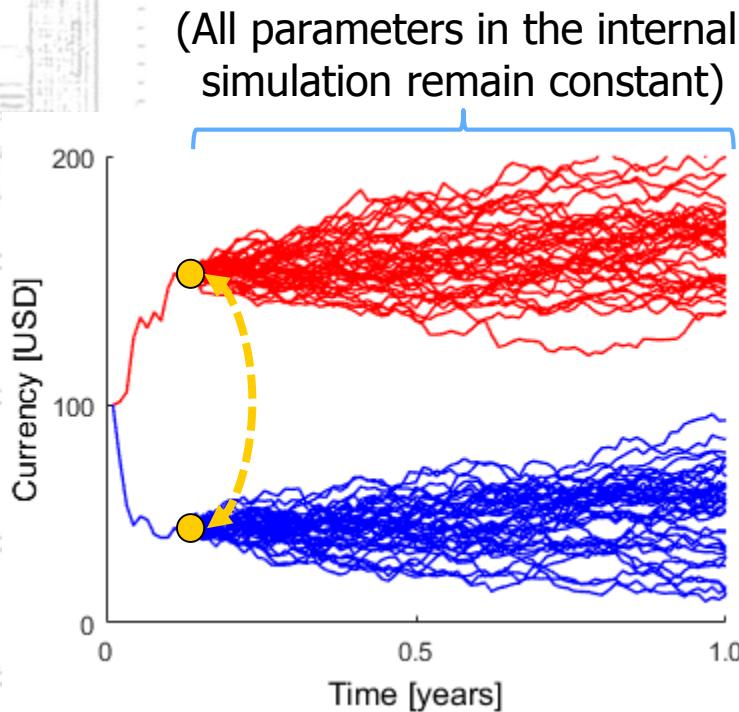
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Algorithmic Optimizations

Random Numbers Reuse Approach

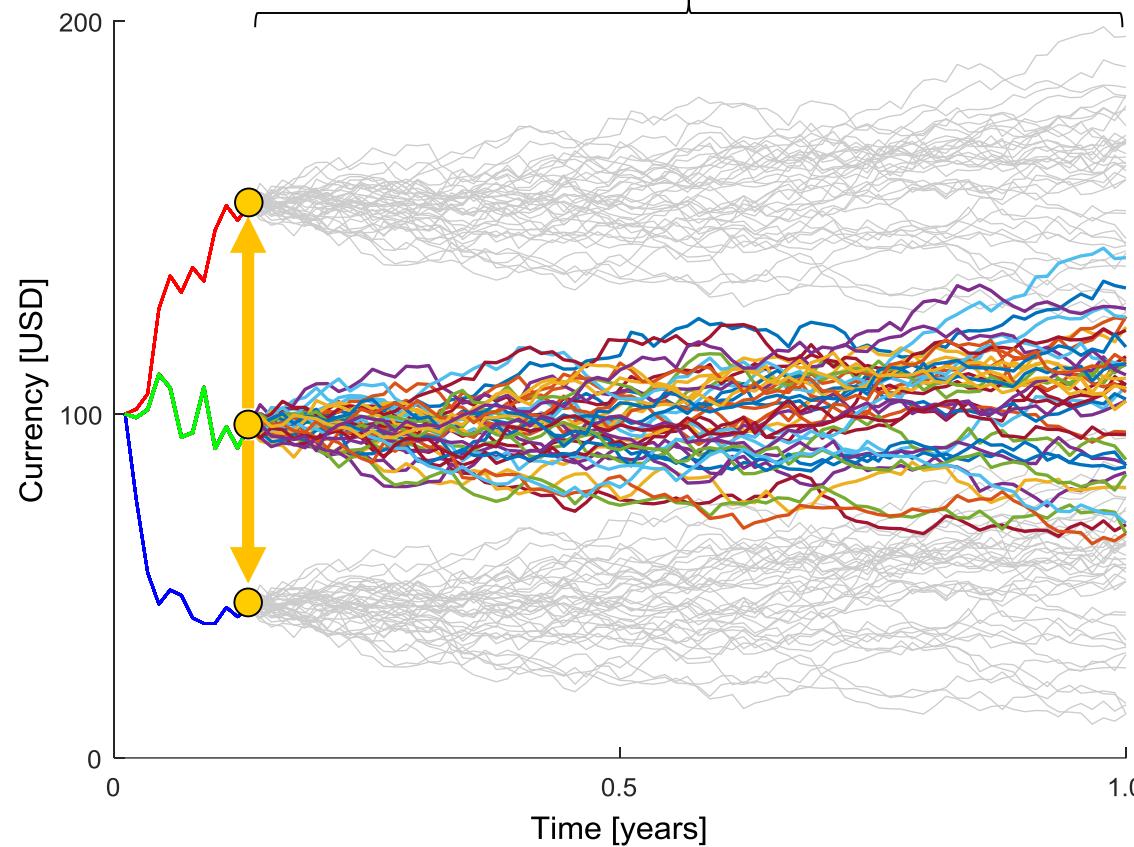
Nested MC-Based Risk Measurement of Complex Portfolios: Acceleration and Energy Efficiency
 S. Desmettre, R. Korn, J. Varela, N. Wehn.
 Risks Vol. 4, no. 4, pages 36, October, 2016.



Algorithmic Optimizations

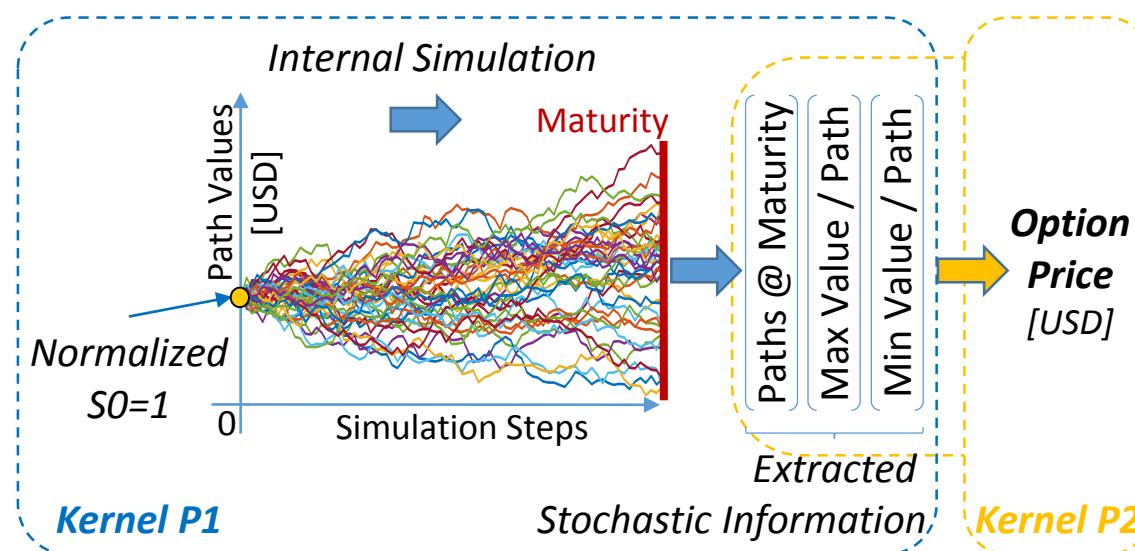
Paths-Reuse Approach (+Normalization)

(Note: All parameters in the internal simulation remain constant)



Algorithmic Optimizations

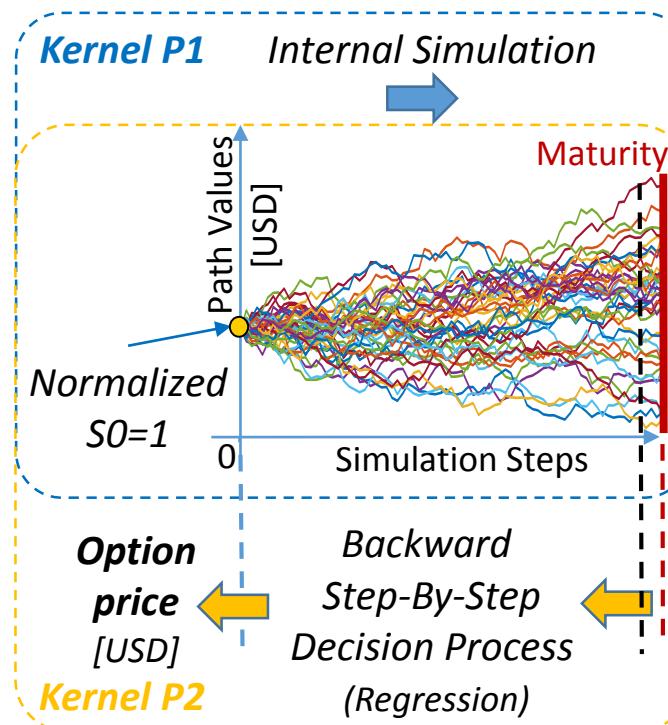
Paths-Reuse: European-style Options



→ **Information-Reuse Approach**

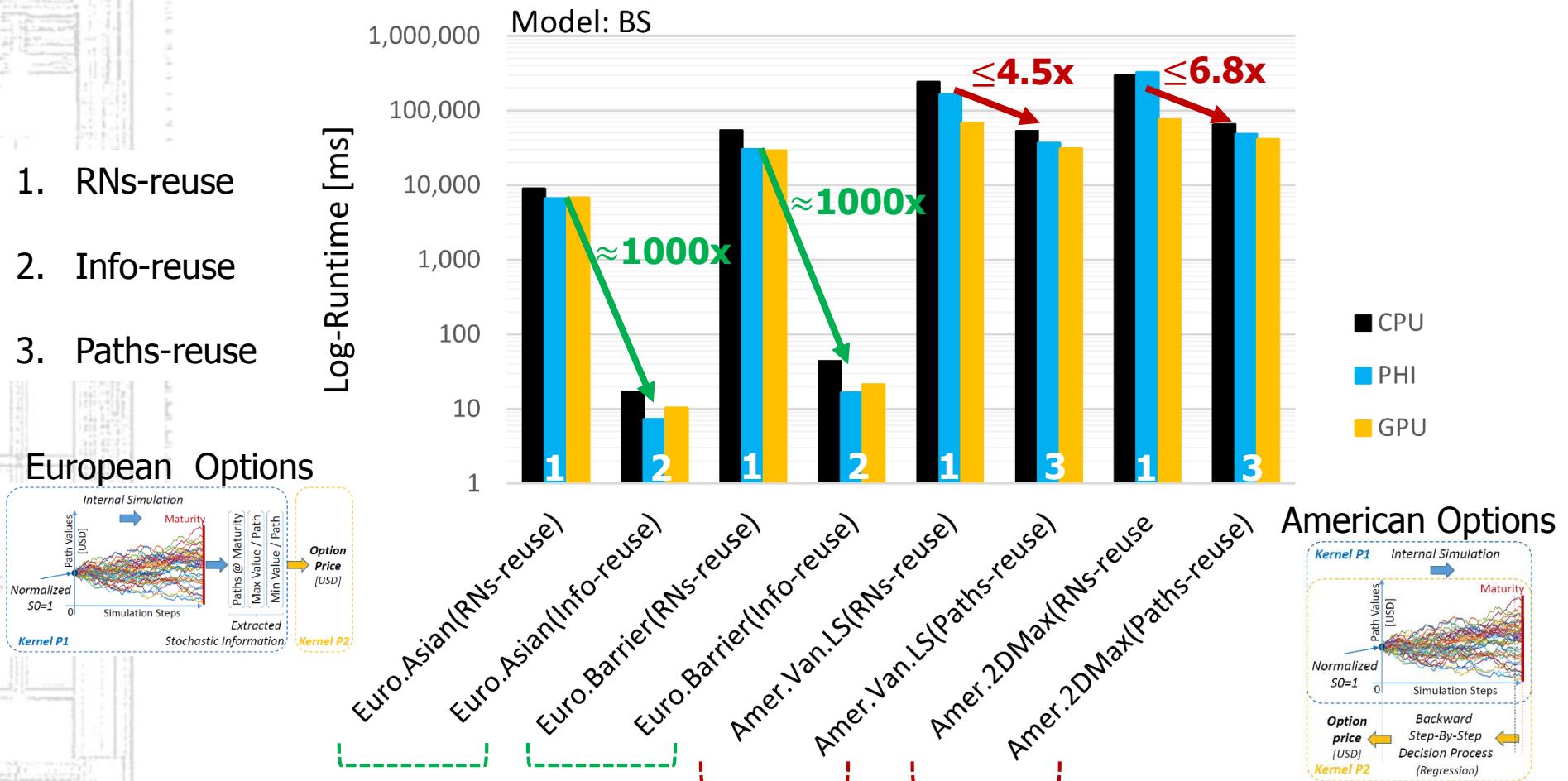
Algorithmic Optimizations

Paths-Reuse: American-style Options



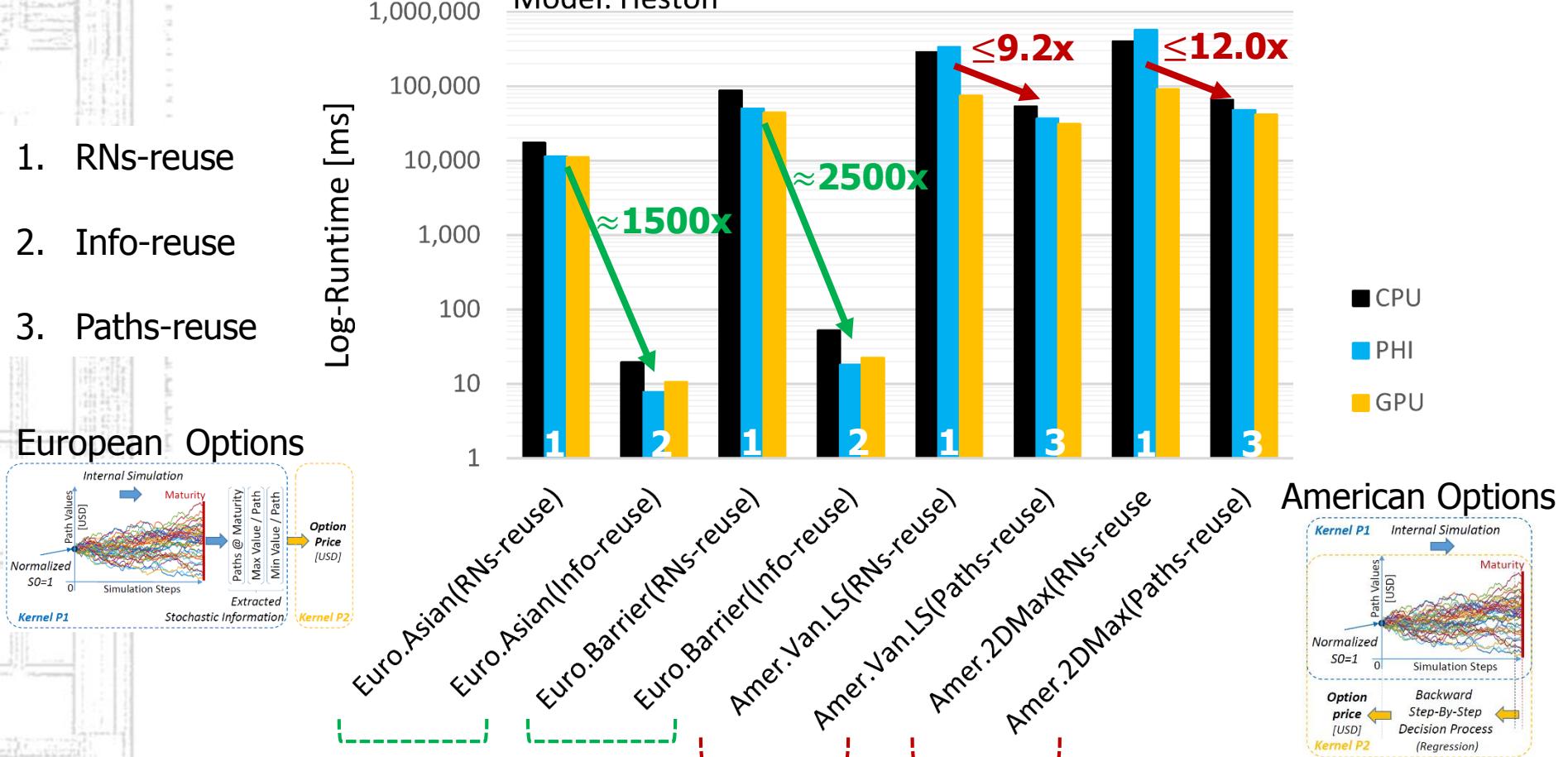
Algorithmic Optimizations

Runtime: Paths-/Info-Reuse (under BS model)



Algorithmic Optimizations

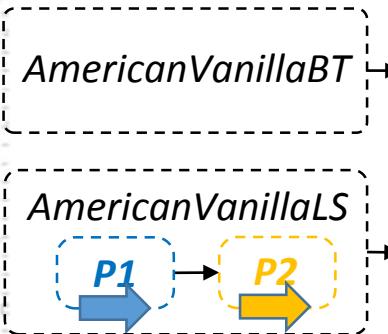
Runtime: Paths-/Info-Reuse (under BS model)



Numerical Scheme

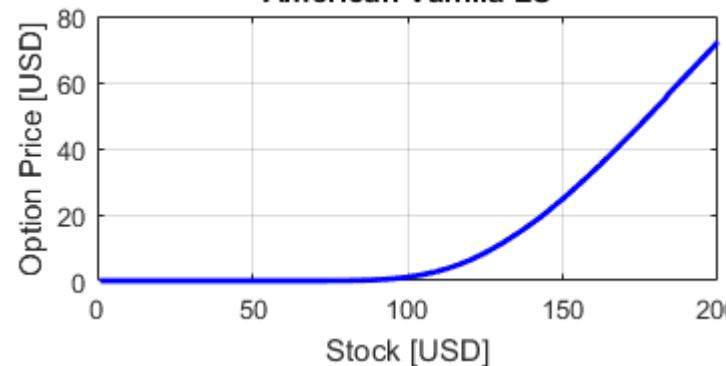
Interpolation:

Kernel:



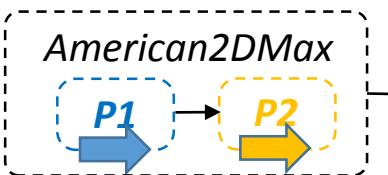
Fast, accurate and inelegant valuation of American options.
 Adriaan Joubert and L.C.G. Rogers. 1997. In Proceedings of the Numerical Methods Workshop at the Isaac Newton Institute, April 1995, L.C.G. Rogers and D. Talay (Eds.). CambridgeS.

American Vanilla LS

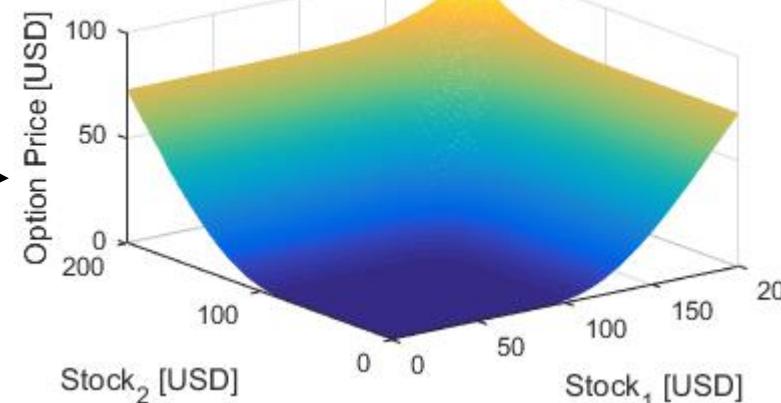


Kernel:
Interpolate 1D
Option Price
 [USD]

Extension:



American 2DMax Call Option

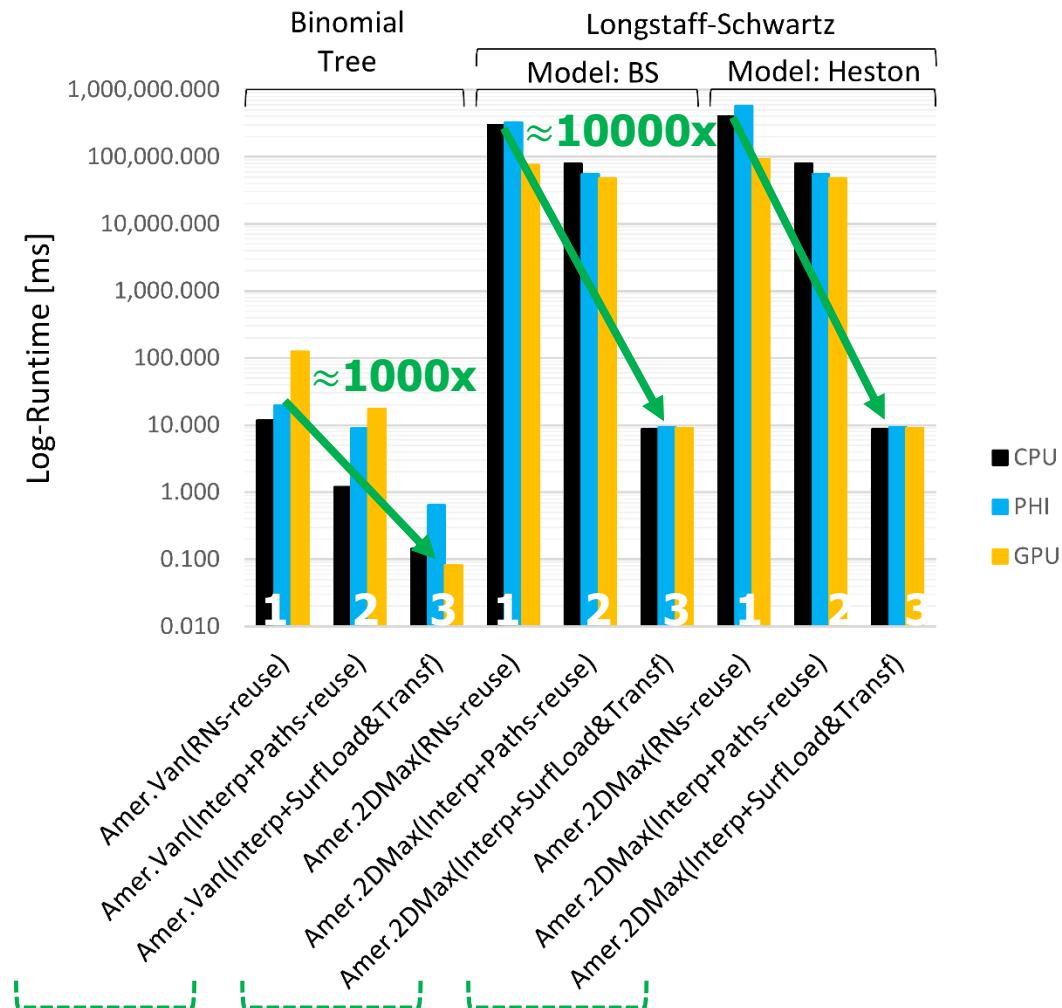


Interpolate 2D
Option Price
 [USD]

Numerical Scheme

Runtime: Interpolation (American-style Options)

1. RNs-reuse
2. Interpolation+Paths-reuse
3. Interpolation+Surface Load&Transfer

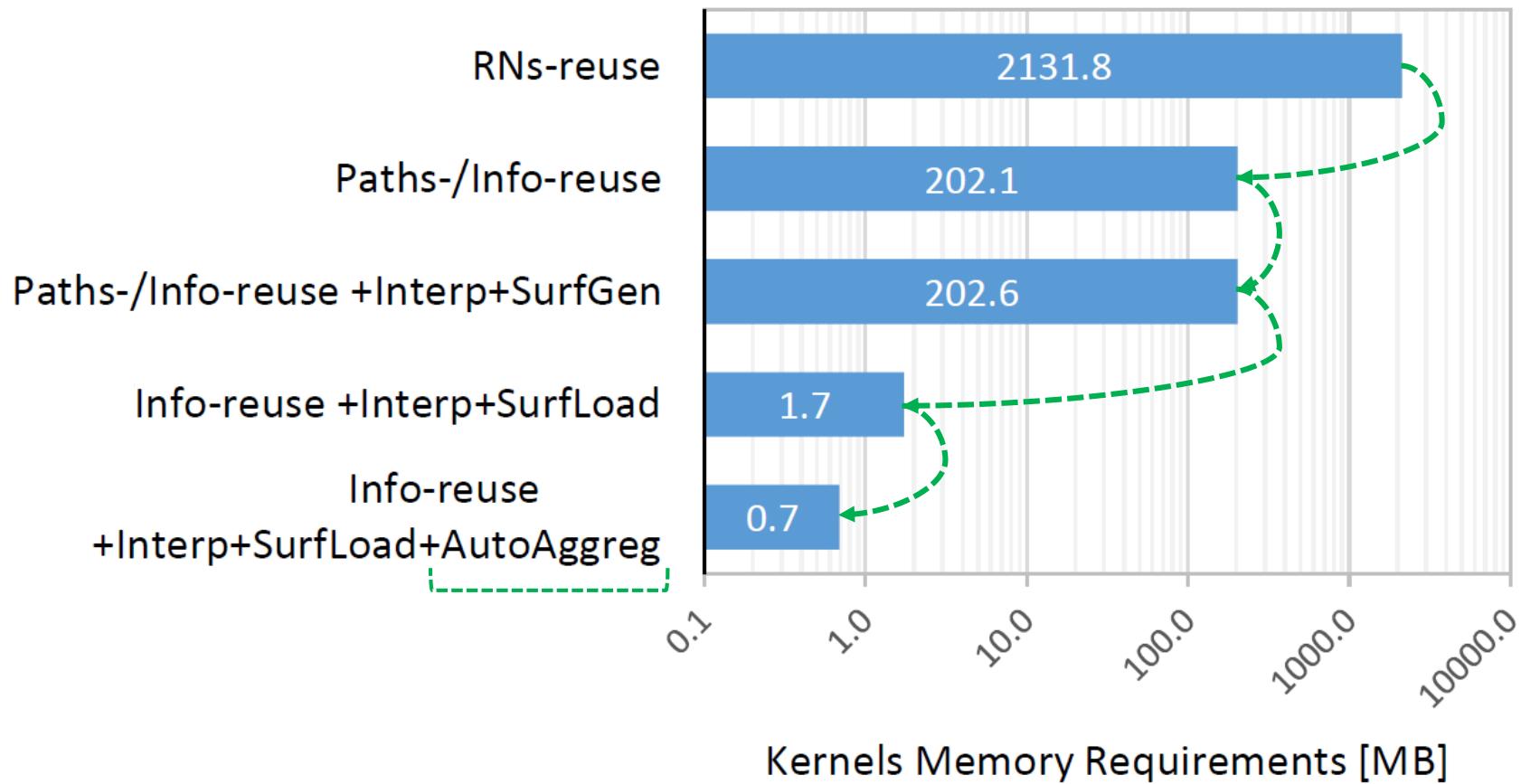


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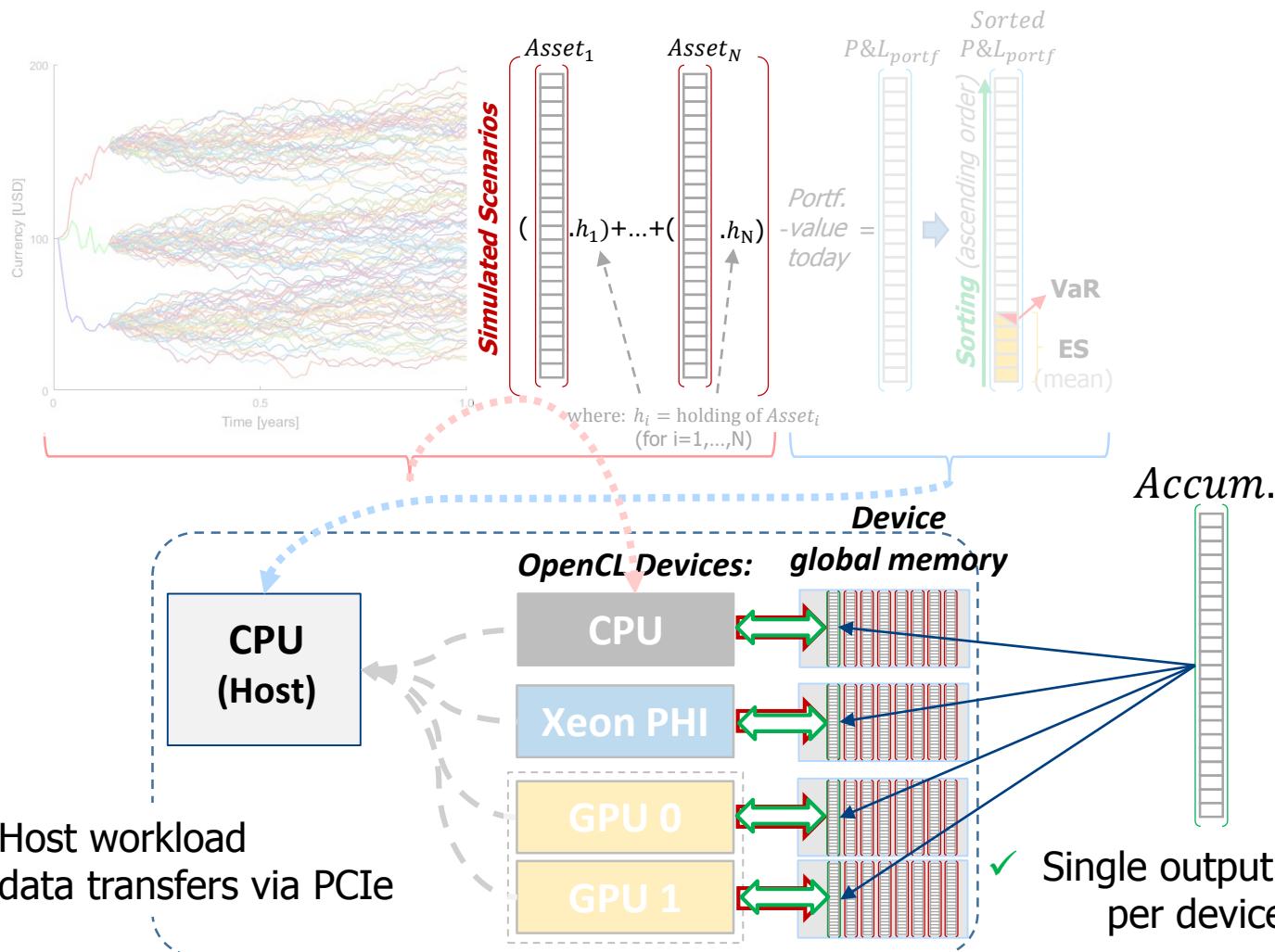
Minimizing Device Global Memory

Device Global Memory Cost (Kernels)



Minimizing Device Global Memory

Autoaggregation

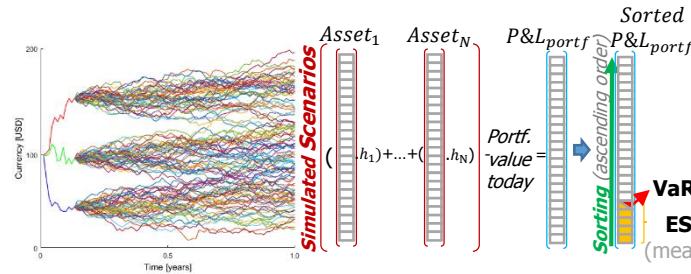


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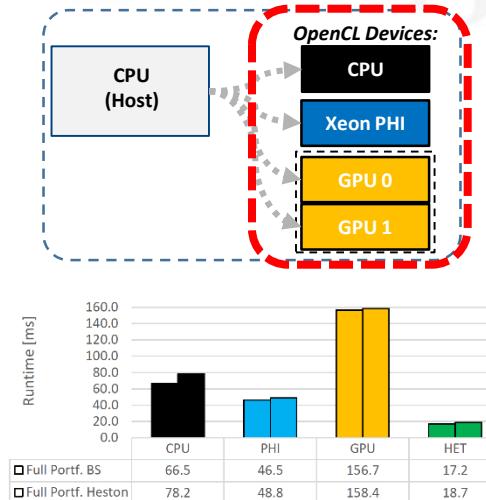
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Conclusion

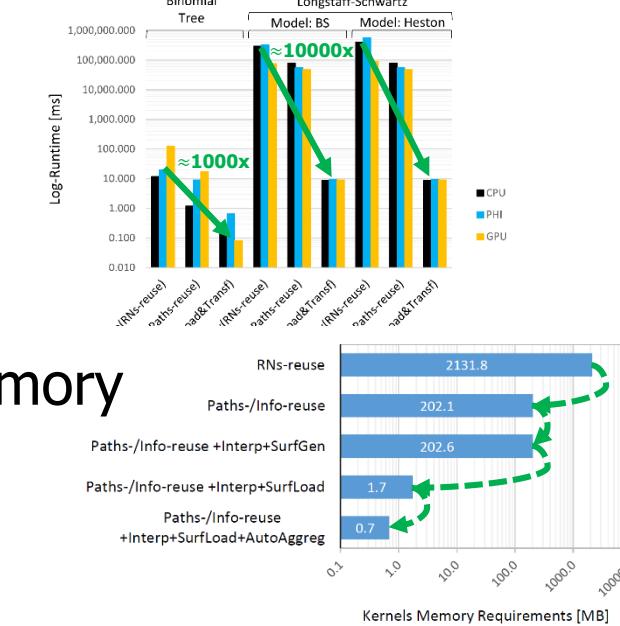
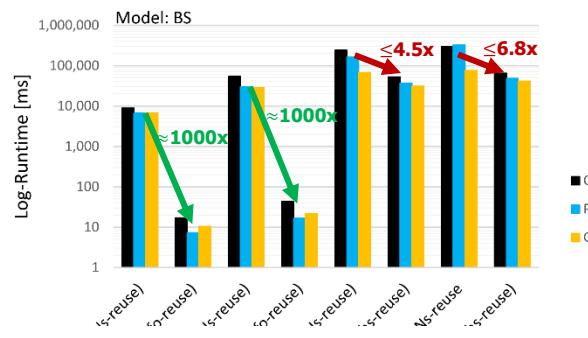
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☐ Portability (Numerical Results)

Questions ?

Thanks for your attention